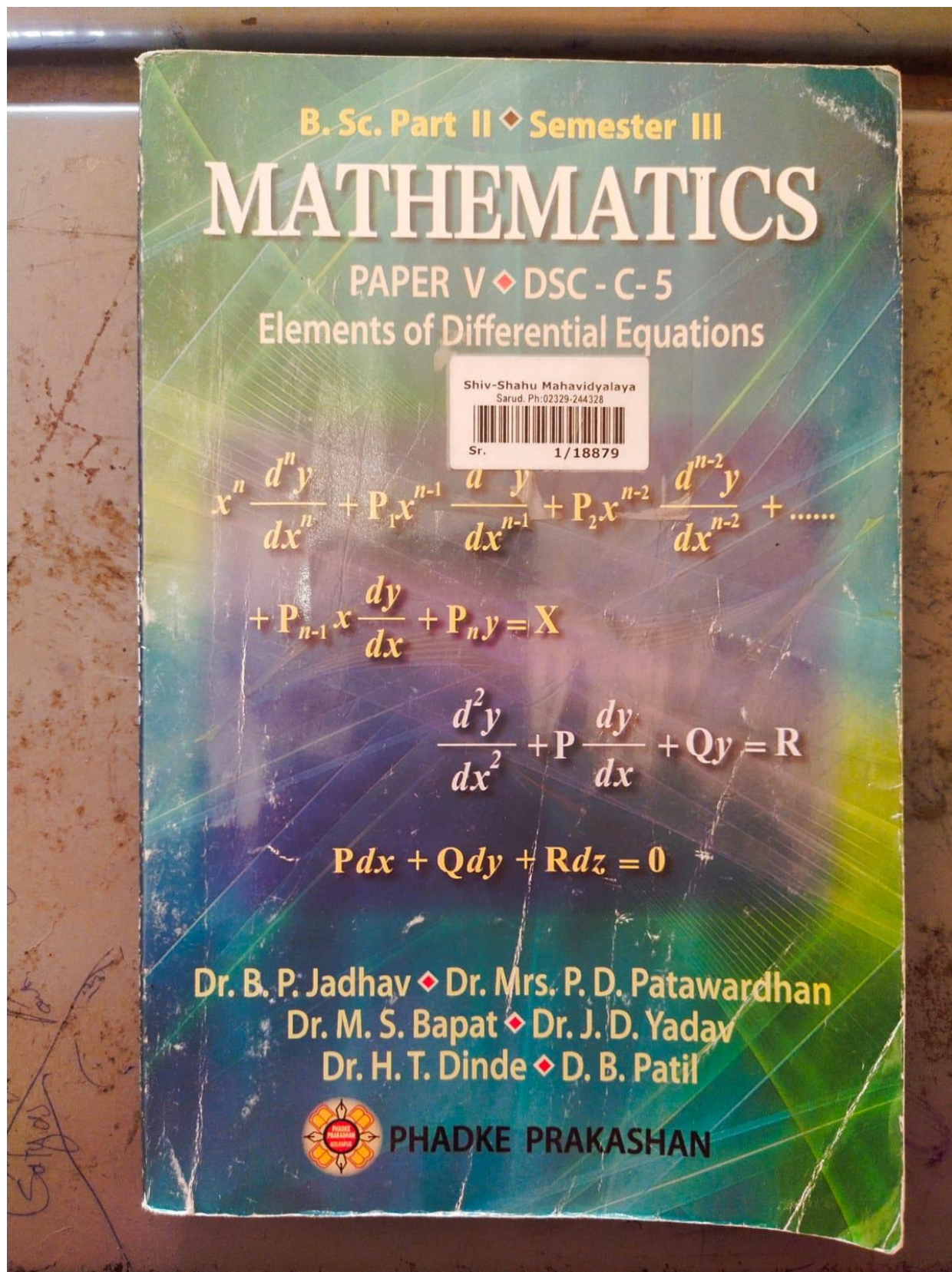


<p>माजी आमदार श्री. बाबासाहेब पाटील सरुडकर शिक्षण संस्था, सरुड</p> <p><b>श्री शिव-शाहू महाविद्यालय, सरुड</b></p> <p>ता. शाहुवाडी, जि.कोल्हापूर</p> <p>(संलग्नीत शिवाजी विद्यापीठ, कोल्हापूर)</p> <p>फोन नं. (०२३२९) २४४३२८</p> <p>ई-मेल आयडी - <a href="mailto:srd56.cl@unishivaji.ac.in">srd56.cl@unishivaji.ac.in</a></p> <p>वेब साईट - <a href="http://www.shrishivshahu.in">www.shrishivshahu.in</a></p> <p>कनिष्ठ महाविद्यालय संकेतांक - जे २३१२००५</p>	<p>Maji Aamadar Shri. Babasaheb Patil Sarudkar Shikshan Sanstha, Sarud</p> <p><b>Shri Shiv-Shahu Mahavidyalaya, Sarud</b></p> <p>Tal.- Shahuwadi, Dist.- Kolhapur</p> <p>(Affiliated to Shivaji University, Kolhapur)</p> <p>Phone No. - (02329) 244328</p> <p>Principal Mob. No. - 9421048948</p> <p>‘B’ Accredited by NAAC, Bengalore</p> <p>Jr. College Code No. J-2312005</p>	
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### 3.3.2 Number of books and chapters in edited volumes/books published and papers published in national/ international conference proceedings per teacher during last five years.

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## Homogeneous Linear Differential Equations

Shiv-Shahu Mahavidyalaya  
Sarud. Ph: 02329-244328



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### 1.1 : Introduction

We already know how to solve a linear differential equations with constant coefficients. In this unit, we are going to solve the *linear differential equations with variable coefficients*.

A linear differential equation with variable coefficients and of the form.

$$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + P_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_{n-1} x \frac{dy}{dx} + P_n y = X \quad \dots (1)$$

where  $P_1, P_2, \dots, P_n$  are constants and  $X$  is a function of  $x$ , is called a **homogeneous linear differential equation**.

### 1.2 : Solution of Homogeneous Linear Equation

The homogeneous linear equation (1) can be reduced to a linear differential equation with constant coefficients by changing the independent variable  $x$  to  $z$  by using substitution.

$$x = e^z, \quad \therefore z = \log x, \quad \frac{dz}{dx} = \frac{1}{x}$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x} \cdot \frac{dy}{dz}$$

$$\text{Let, } \frac{d}{dz} = D, \quad \therefore x \frac{dy}{dx} = \frac{dy}{dz} = Dy \quad \dots (2)$$

$$\begin{aligned} \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} \left( \frac{1}{x} \frac{dy}{dz} \right) \\ &= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \frac{d}{dx} \left( \frac{dy}{dz} \right) \\ &= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \cdot \frac{d}{dz} \left( \frac{dy}{dz} \right) \cdot \frac{dz}{dx} \end{aligned}$$

Advances in Geographical and Environmental Sciences


R. B. Singh  
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# Remote Sensing and Geographic Information Systems for Policy Decision Support



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## Chapter 4

# Site Suitability Analysis for Identifying Water Conservation Structures Using Geoinformatics of Eastern Part of Satara District of Maharashtra, India



P. T. Waghmare and S. S. Panhalkar

**Abstract** Identification of a suitable site for water harvesting structure is significant for drought mitigation and management. Water conservation needs an in-depth study of rainfall-runoff features and a thorough assessment of surface topographical conditions. In this study, the researcher has identified appropriate site for water harvesting structures in the eastern part of Satara district of Maharashtra. The present investigation uses physical components as well as social components. Remote sensing data and toposheets and integrated weighted overlay methods are used to identification of proper suitable sites for water harvesting system. The minimum value was assigned to the factor that is least suitable for the water harvesting system and the maximum value was given to the factor that is highly favorable for the water harvesting structure. The influence factor values of thematic layers were summed up and the total score was calculated. Finally, the calculated score was classified into four classes. The investigation brings out that 31.15% area is highly suitable for establishing water harvesting structures. Moderately suitable area is about 45.65, whereas less suitable area is 20% for water conservation structures. The present study will be helpful to reduce the risk of future drought conditions.

**Keywords** Runoff · Weighted overlay analysis · SCS-CN method · GIS · Remote sensing

## 4.1 Introduction

Water is an essential component for fulfilling the basic needs of biotic and abiotic components. Water scarcity has increased due to quickly rising population, industry, economy, and climate change; managing water resources has always been a difficult task during the drought years in past decades (Hirsch 1981; Frick et al. 1990; Randall et al. 1990; Johnson and Kohne 1995; Smithers 1997). Due to over-exploitation and inadequate natural recharge, groundwater table in the study area is going

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
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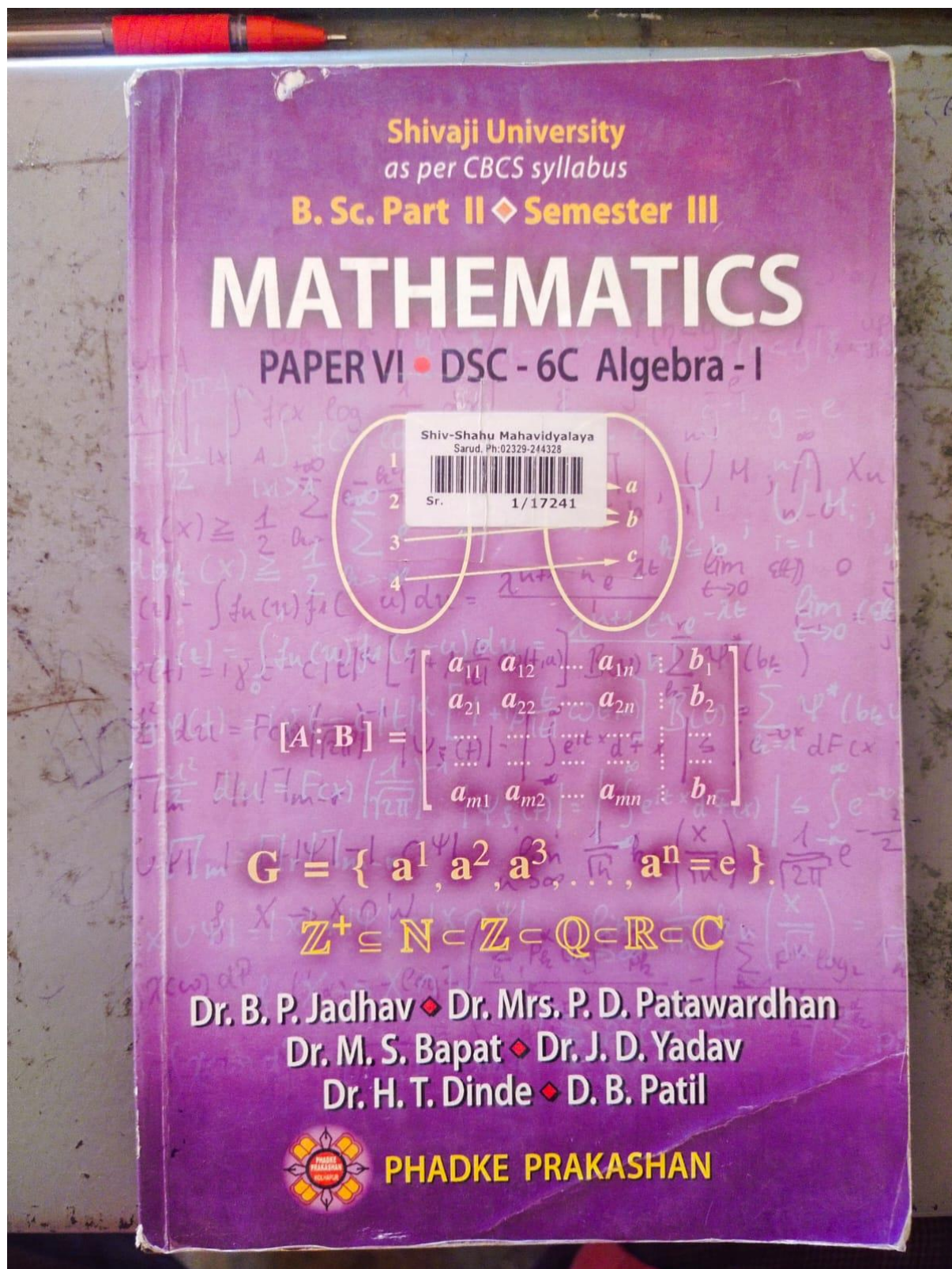
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Dr. H.T.Dinde



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CHAPTER

1

# MATRICES

Shiv-Shahu Mahavidyalaya  
Sarud. Ph: 02329-244328



Sr. 1/17241

## 1.1 : Introduction

Matrix theory acts as a powerful tool in almost all branches of social and physical sciences. Matrices are used in engineering, linear programming, statistics, economics and also in industrial management. The subject of matrices has its origin in the solution of "linear problems". Cayley formulated the general theory of matrices. In standard XI and XII we have already studied some basic concepts of matrices. In this chapter, we shall study Hermitian and Skew-Hermitian matrices, system of linear homogeneous equations and system of linear non-homogeneous equations, eigenvalues and eigenvectors of matrices, Cayley-Hamilton theorem.

## 1.2 : Some Basic Definitions

First we shall revise some basic definitions:

**Definition :** A system of  $mn$  numbers (real or complex) arranged in the form of an ordered set of  $m$  rows, each row consisting of an ordered set of  $n$  numbers is called a matrix of order  $m$  by  $n$ , written as  $m \times n$  matrix.

**Definition :** A square matrix is said to be a **triangular matrix** if the elements above or below the principal diagonal are zero.

**Definition :** A square matrix  $A = [a_{ij}]$  is called an **upper triangular matrix**, if  $a_{ij} = 0$ , for  $i > j$ .

**Definition :** A square matrix  $A = [a_{ij}]$  is called a **lower triangular matrix**, if  $a_{ij} = 0$ , for  $i < j$ .

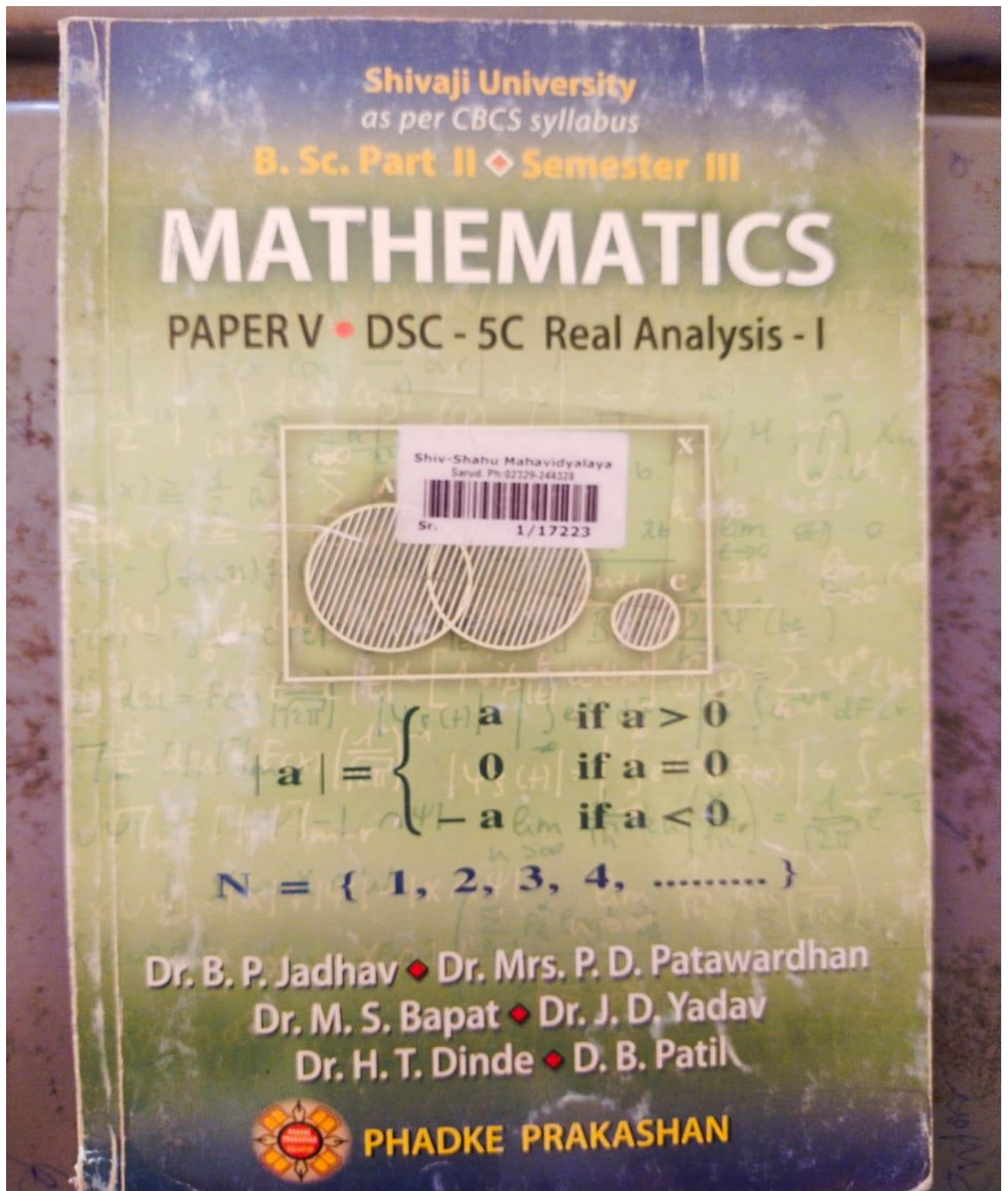
**Definition :** A square matrix  $A$  is said to be singular or non-singular according as  $|A| = 0$  or  $|A| \neq 0$ .

**Definition :** A square matrix  $A = [a_{ij}]$  is called symmetric if  $a_{ij} = a_{ji}$  for all  $i$  and  $j$ .  $A^T = A$

**Definition :** A square matrix  $A = [a_{ij}]$  is called skew-symmetric if  $a_{ij} = -a_{ji}$  for all  $i$  and  $j$ .  $A^T = -A$

**Note :** All the diagonal elements of a skew-symmetric matrix are zero.

Dr. H.T.Dinde



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Shiv-Shahu Mahavidyalaya  
Sarud. Ph. 02329-244328

**UNIT-I**

Sr. 1/17223

<b>CHAPTER</b>	1	Sets and Functions

### 1.1 Introduction

The theory of sets can be regarded as the foundation of modern mathematics. Set theory was founded by German Mathematician G. Cantor (1845-1918). Algebra and Analysis were formulated within a framework of set theory. A set is an intuitive notion on which all other notions will be based. It has been experienced that it is very difficult to define a set. The Mathematicians realised that there must be some undefined (or primitive) terms. Here we start with two undefined terms "element" and "set". By an element we mean an object or entity of some sort *e.g.* a point on the real line (= a real number). A set is a well-defined collection of objects taken as a whole. The words class, collection, family are synonyms of the word set. Given an object and a set we can say whether the object belongs to the set or does not belong to the set.

The idea of set is not new to us as it is commonly used in everyday life. *e.g.* (i) a bundle of news-papers (ii) a group of students.

Sets are usually denoted by capital letters A, B, C, X, Y..... etc. If  $x$  is an element of a set S, then we write this as  $x \in S$  ( $x$  belongs to S) and if an object  $y$  is not an element of a set S, then we write it as  $y \notin S$  ( $y$  does not belong to S)

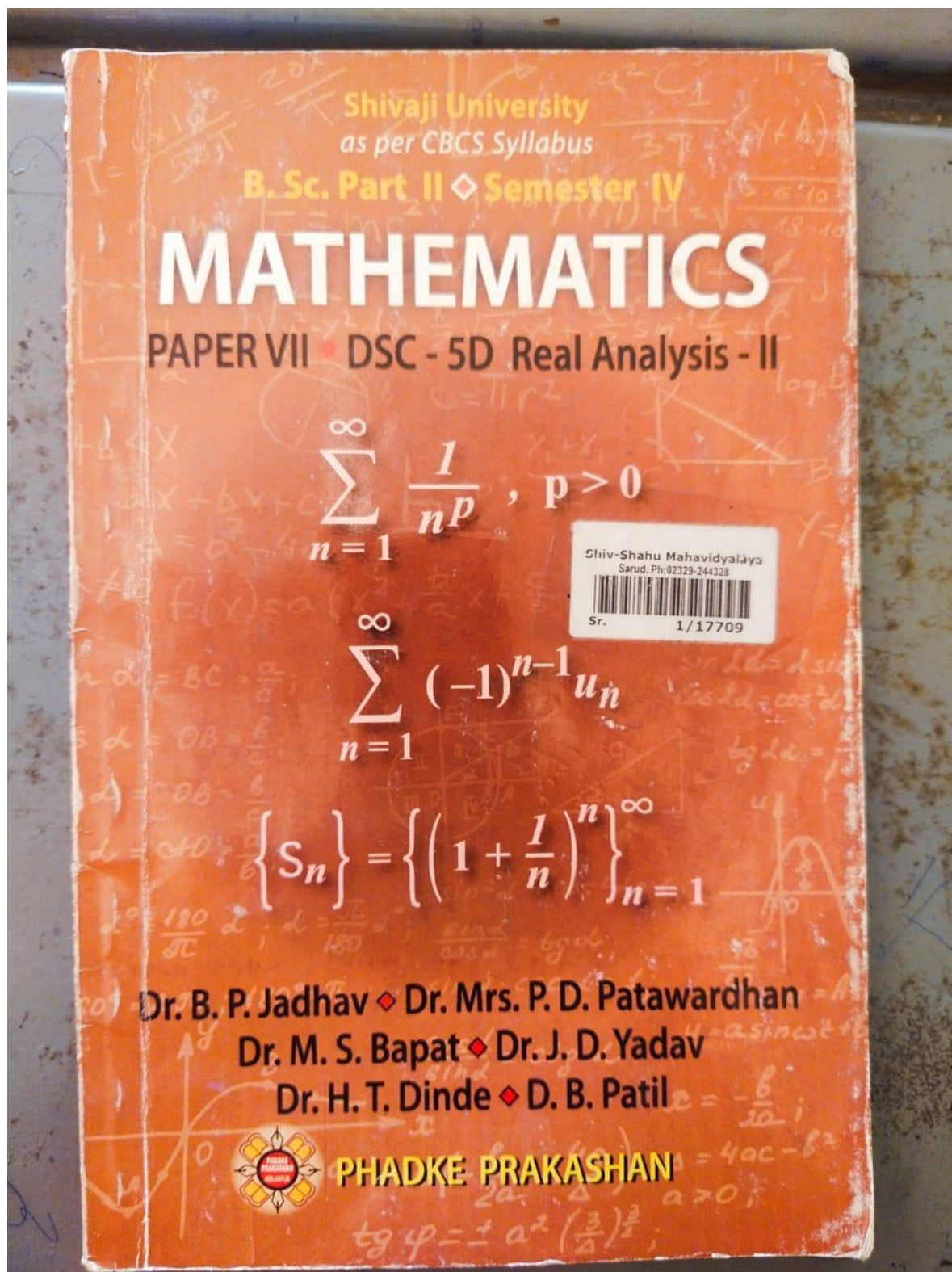
If we write  $A = \{1, 3, 5, 7, 9\}$  then  $5 \in A$ ,  $9 \in A$  but  $10 \notin A$ .

### 1.2 Revision of Sets

#### Finite Set and Infinite Set

If we start counting the number of elements in a given set and if this process of counting comes to an end at some stage, then the set is said to be finite set. If this process of counting does not stop at any stage then it is called an infinite set.

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UNIT

CHAPTER

1

1.1 : Sequence

Definition :

and co-domain is

Sequence is

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(b) {

(c)

(d)

(e)

1.2 : Subseq

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Solution : C

Shivaji Uni.

UNIT 1 : Sequence of Real Numbers

CHAPTER

1

Sequences and Operations on Sequences

1.1 : Sequence

**Definition :** Sequence is a function whose domain is set of natural numbers and co-domain is set of real numbers.

Sequence is usually denoted by  $\{S_n\}_{n=1}^{\infty}$  or  $\langle S_n \rangle_{n=1}^{\infty}$  or simply  $\{S_n\}$  and  $n^{\text{th}}$  term of sequence by  $S_n$ .

e.g., (a)  $\{S_n\} = \{n^2\} = \{1, 4, 9, 25, \dots, n^2, \dots\}$

(b)  $\{S_n\} = \left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots\right\}$

(c)  $\{S_n\} = \left\{\frac{n}{n+1}\right\} = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, \frac{n}{n+1}, \dots\right\}$

(d)  $\{S_n\} = \{(-1)^n\} = \{-1, 1, -1, 1, -1, \dots, (-1)^n, \dots\}$

(e)  $\{S_n\} = \{1\} = \{1, 1, 1, 1, \dots, 1, \dots\}$

1.2 : Subsequence of Sequence

**Definition :** Let  $S = \{S_n\}$  be sequence of real numbers and  $N = \{n_i\}_{i=1}^{\infty}$  be subsequence of set of natural numbers, then the composite function  $S \circ N$  is called subsequence of  $S$  and defined as

$$S \circ N (i) = S [N (i)] = S (n_i) = S_{n_i}$$

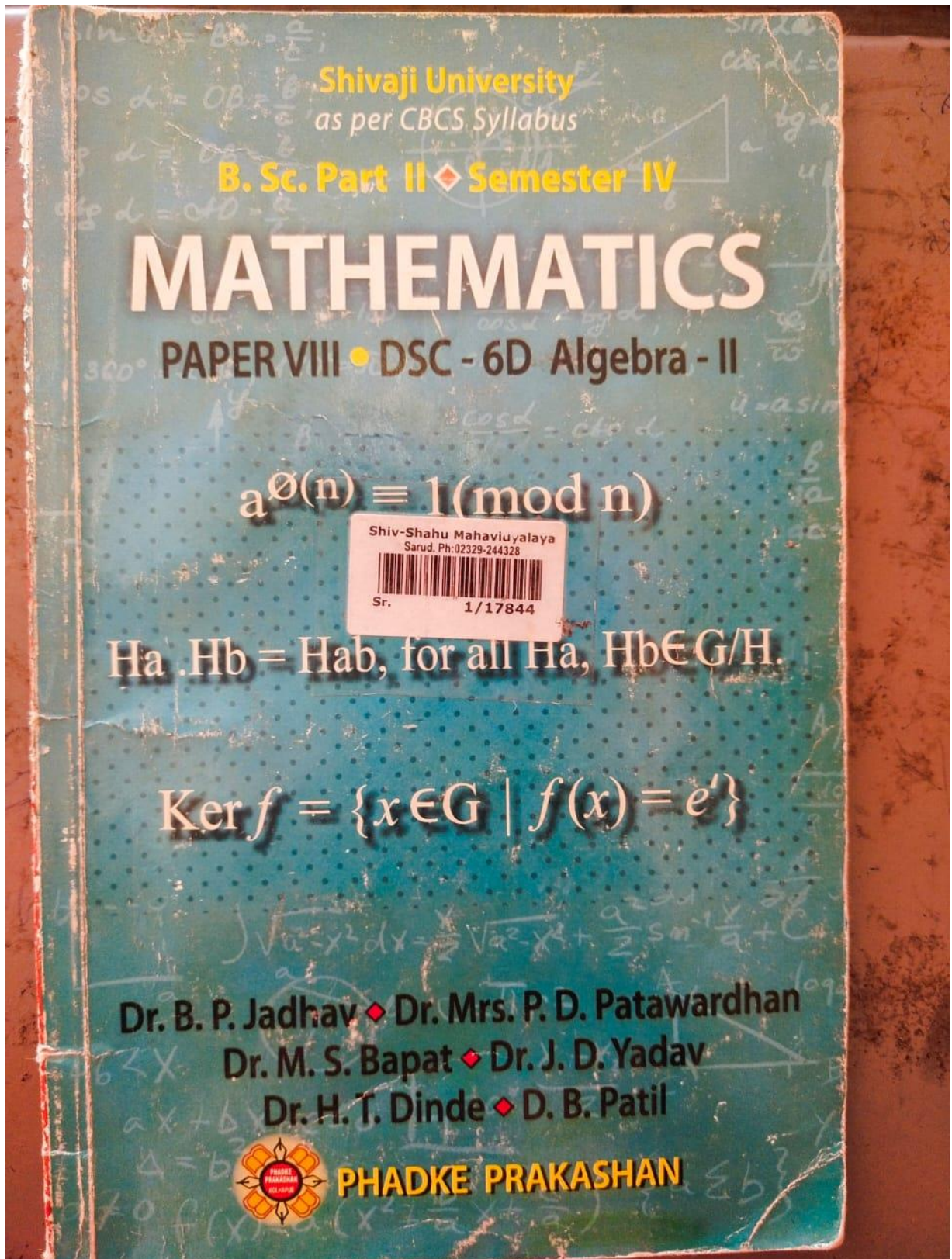
$$\text{Hence, } S \circ N = \{S_{n_i}\}_{i=1}^{\infty}$$

Illustrative Examples

**Example 1 :** If  $S = \{S_n\} = \{n^2\}_{n=1}^{\infty}$  and  $N = \{n_i\} = \{2i-1\}, i \in N$ , then find  $S \circ N$ .

**Solution :** Given :  $S = \{S_n\} = \{n^2\}_{n=1}^{\infty}$

and  $N = \{n_i\} = \{2i-1\}, i \in N$



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We have great pleasure in publishing this book for B. Sc. Part-II (Semester II) of Kolhapur University, Kolhapur.

This textbook is brought in accordance with the syllabus of the University.

Special effort has been made to cover all the topics and present the material in a simple and lucid manner. Nowadays Mathematics is in the hands of all. "Mathematics is the language of Science" and "Mathematics is the language of Mathematics".

The salient features of the book are:

- ♦ Topics are arranged in a logical sequence.
- ♦ Syllabus (CBCS) is followed.
- ♦ Simple and lucid language is used.
- ♦ Illustrations are given to help understanding.
- ♦ Variety of exercises are given.
- ♦ Miscellaneous exercises are given at the end of each topic.
- ♦ Multiple choice questions are given at the end of each chapter.
- ♦ Figures are given wherever necessary.

We are thankful to the authorities of Kolhapur University, Kolhapur for their excellent type-setting and for their valuable suggestions.

Any suggestion for improvement in the book is welcome.

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## CHAPTER

# 1

### 1.1 : Introduction

We know that no subset  $H$  of finite group  $G$  has some relation between the order of a subgroup  $H$  and the order of  $G$ . This is called Lagrange's Theorem. Mathematician Joseph Lagrange actually proved the theorem. It actually proved by Cauchy. It plays a key role in the study of groups.

### 1.2 : Lagrange's Theorem

**Theorem 1 :** If  $H$  is a subgroup of  $G$ , then  $|H|$  divides  $|G|$ .

**Proof :** Let  $G$  be a finite group.

Let  $H$  be a subgroup of  $G$ .

(i) Suppose  $H = \{e\}$ .

If  $H = \{e\}$ , then  $|H| = 1$ .

If  $H = G$ , then  $|H| = |G|$ .

Thus if  $H$  is a subgroup of  $G$ , then  $|H|$  divides  $|G|$ .

(ii) Suppose  $H \neq \{e\}$  and  $H \neq G$ .

Let  $o(H) = n$ .

Since  $G$  is a finite group,  $|G|$  is finite.

Let  $H$  be a subgroup of  $G$ .

Let  $H_{a_1}, H_{a_2}, \dots, H_{a_r}$  be the cosets of  $H$  in  $G$ .

Shivaji Uni. ► B.Sc. I

